Influence of turbulence intensity and free stream velocity oscillations on stagnation point heat transfer

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A model is proposed for the momentum eddy diffusivity induced by free stream turbulence intensity and integral length scale. The eddy diffusivity model is applied to the stagnation point of a cylinder situated in a steady uniform crossflow in the presence of free stream turbulence. A numerical solution of the governing steady-state momentum and energy equations with the proposed eddy diffusivity model yielded results for the skin friction coefficient and the Nusselt number. Agreement between the numerical predictions of this work and experimental data is very good. The experimental data concerning the unsteady stagnation point heat transfer under the combined influence of free stream velocity oscillations and turbulence intensity have been successfully correlated by means of a new turbulence parameter

Key words: *heat transfer, turbulence, velocity, stagnation point*

It is known that free stream turbulence augments significantly the rate of heat transfer in flows with large streamwise pressure gradients. Several experimental and analytical studies[1-10] have been conducted to examine the free stream turbulence effects on heat transfer.

Sutera *et al*[11,12] simulated the free stream turbulence by a distributed vorticity in the free stream. Their model assumed a vorticity component in the oncoming flow to be unidirectional and oriented so that the vortex lines were susceptible to stretching. They found that amplification by stretching of vorticity of sufficiently large scale can occur. As a result, such vorticity, with small intensity in a flow, can appear near the boundary layer with an amplified intensity. They also found that the thermal boundary layer is apparently much more sensitive to the induced effects than is the velocity boundary layer. Further elucidation of the mechanism by which the presence of the vortices affects the heat transfer has been provided by Kestin and Wood[13-15].

Smith and Kuethe[6] postulated an expression for the eddy diffusivity induced by the free stream turbulence. Their study revealed that the measured values near the stagnation point on a circular cylinder were increased by about 70% for heat transfer and 50% for skin friction with a free stream turbulence level of 6%. Galloway[16] proposed a model on the assumption that the heat transfer in the presence of free stream turbulence is enhanced by Goertler vortices. Traci and Wilcox[17] presented a two-equation model of turbulence. They evaluated the Nusselt number and friction factor at the stagnation point of a cylinder situated in crossflow.

One of the most significant studies of laminar boundary layers under the influence of a purely time-dependent, free stream oscillation has been carried out by Lighthill[18]. He showed that various layers in the boundary layer acquire oscillations which experienced phase shift and that the fluctuation amplitude decreased away from the wall. Recently, Gorla, Jankowski and Textor[19] investigated the time-mean characteristics of the laminar boundary layer near an axisymmetric stagnation point when the velocity of the oncoming flow relative to the body oscillates. Different solutions were obtained for small and high values of the reduced frequency parameter. The authors presented numerical solutions for the velocity and temperature fields. Base *et al*[20] conducted experiments to determine the forced convective heat transfer from the stagnation point of heated cylinders when the approaching flow has low frequency oscillations.

Scope of the present work

The above review of the published literature suggests that no analyses have been reported which study the effect of the integral length scale of the free stream turbulence on the steady-state stagnation point heat transfer. It is also found from the literature survey that no correlations are reported to determine the combined effects of free stream velocity oscillations and free stream turbulence intensity on the

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unsteady heat transfer from a stagnation point.

The aims of this study were:

(i) Formulation of a model for the eddy diffusivity induced by free stream turbulence intensity and integral length scale. The general eddy diffusivity model has been applied at the stagnation point of a cylinder in a uniform crossflow and the Nusselt number and friction factor have been evaluated as functions of the free stream turbulence intensity and integral length scale by solving the appropriate governing equations under steady-state conditions. This basic study facilitates understanding of the complex heat transfer phenomena in the real turbomachinery environment to a first degree of approximation.

(ii) Correlation of the experimental data for transient heat transfer from a stagnation point under the combined influence of velocity oscillations and turbulence in the free stream.

Analysis of steady-state heat transfer and eddy diffusivity formulation

Assuming a steady, incompressible laminar flow with constant properties and negligible dissipation, the governing boundary-layer equations in the vicinity of the stagnation point on a cylinder in a uniform crossflow may be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_e\frac{\mathrm{d}U_e}{\mathrm{d}x}+\frac{\partial}{\partial y}(\tau/\rho) \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{-\partial}{\partial y}(q/\rho c_p) \qquad (3)$$

In the above equations, x and y represent the distances along the streamwise direction and normal direction; u and v the velocity components in the x and y directions; U_e the external velocity; ρ the density of the fluid; T the temperature, c_p the specific heat of the fluid; τ the shear stress and q the heat flux.

The appropriate boundary conditions may be written as:

$$y=0: \quad u=v=0 \quad \text{and} \quad T=T_w$$
$$y\to\infty: \quad u\to U_e \quad \text{and} \quad T\to T_\infty \qquad (4)$$

The shear stress and the heat flux in Eqs (2) and (3) may be written as:

$$\tau=\rho(\nu+\varepsilon_m)\frac{\partial u}{\partial y}$$
$$q=-\rho c_p\left(\alpha+\frac{\varepsilon_m}{Pr_t}\right)\frac{\partial T}{\partial y} \qquad (5)$$

This paper is restricted to the case of homogeneous and isotropic free stream turbulence. This will be realized in the turbulence produced by grids at locations sufficiently downstream from the turbulence promoters. The eddy diffusivity has been assumed to be:

$$\frac{\varepsilon_m}{\nu}=s(\eta+\eta^2)\mathscr{F}(\bar{L}) \qquad (6)$$

where $\bar{L}$ is the dimensionless integral length scale of free stream turbulence, s is a constant proportional to the turbulence intensity and η is the dimensionless distance normal to the surface.

More discussion of the eddy diffusivity model outlined in Eq (6) will be provided in later sections of the paper.

Proceeding with the analysis, a stream function ψ is defined such that $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$. It may be verified that the continuity equation is automatically satisfied. Further defined are:

$$\eta=y\left(\frac{K}{\nu}\right)^{1/2}$$
$$\psi=(\nu K)^{1/2}xF(\eta)$$
$$u=KxF'(\eta) \qquad (7)$$
$$\theta=(T-T_\infty)/(T_w-T_\infty)$$

Notation

c_f	Skin friction coefficient $\tau_w/(\frac{1}{2}\rho U_\infty^2)$
c_p	Specific heat
D	Diameter of cylinder
f	Frequency of oscillations of free stream velocity
$\hat{f}$	Strouhal number (fD/U_∞)
K	Thermal conductivity
L_x	Integral length scale of free stream turbulence
$\bar{L}$	Dimensionless integral length scale defined in Eq (13)
Nu	Nusselt number (hD/k)
Pr_t	Turbulent Prandtl number ($\varepsilon_m/\varepsilon_h$)
q	Heat flux rate
Re	Reynolds number ($U_\infty D/\nu$)
T	Temperature
Tu	Turbulence intensity
t	Time
u, v	Velocity components
U_e	Free stream velocity
x, y	Coordinates
α	Thermal diffusivity
$\varepsilon_m, \varepsilon_h$	Momentum and thermal eddy diffusivities respectively
η	Dimensionless distance
ρ	Density
τ_w	Wall shear stress

Subscripts

w	Wall conditions
∞	Ambient conditions

The prime above denotes differentiation with respect to η. Upon substituting expressions in Eq (7) into Eqs (2) and (3), the transformed momentum and energy equations may be written as follows:

$$\left[\left(1+\frac{\varepsilon_m}{\nu}\right)F''\right]'+FF''+[1-(F')^2]=0 \qquad (8)$$

$$\left[\left(\frac{1}{Pr}+\frac{1}{Pr_t}\frac{\varepsilon_m}{\nu}\right)\theta'\right]'+F\theta'=0 \qquad (9)$$

The transformed boundary conditions are:

$$F(0)=F'(0)=0 \quad \text{and} \quad F'(\infty)=1$$
$$\theta(0)=1 \quad \text{and} \quad \theta(\infty)=0 \qquad (10)$$

Once Eqs (8) and (9) are solved, the skin friction coefficient c_f and the Nusselt number Nu may be calculated as follows:

$$c_f=\frac{\tau_w}{(\frac{1}{2}\rho U_\infty^2)}=13.838\frac{x}{D}Re^{-1/2}F''(0)$$
$$Nu=-1.905518Re^{1/2}\theta'(0) \qquad (11)$$

The Prandtl number Pr is 0.72 for air at 20 °C and it is assumed that the conventionally-used value of the turbulence Prandtl number $Pr_t=0.9$ is also applicable to the present problem.

Eqs (8) and (9) are solved on a computer using the fourth order, Runge–Kutta numerical procedure. Double precision arithmetic was used in all the computations. The selection of the integration step size depends on the accuracy desired. After some computational trials, a step size of $\Delta\eta=0.001$ was chosen.

Correlation of experimental data for transient heat transfer

Many modern turbine blade sections, even at the high Reynolds numbers at which they operate, manifest considerable areas over which the boundary layer remains laminar in a steady, low turbulence stream. The superimposition of mainstream velocity fluctuations associated with artificially introduced mainstream turbulence is known to affect dramatically heat transfer rates under such conditions. Such effects can be important in turbine blade design for, as rotor speed changes, so will the frequency of the perturbations in flow velocity which the blades experience.

A brief discussion will now be provided for the governing equations of the unsteady problem. Assuming an incompressible flow with constant properties and negligible dissipation, the governing equations, within the framework of the boundary-layer approximation, may be written as:

$$\text{Mass} \qquad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (12)$$

$$\text{Momentum:} \quad \frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial U_e}{\partial t}+U_e\frac{\partial U_e}{\partial x}+\frac{1}{\rho}\frac{\partial \tau}{\partial y} \qquad (13)$$

$$\text{Energy:} \quad \frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=-\frac{1}{\rho c_p}\frac{\partial q}{\partial y} \qquad (14)$$

The boundary conditions for the wall and for the outer edge of the boundary layer are:

$$y=0, \quad u=v=0 \quad \text{and} \quad T=T_w$$
$$y\to\infty: \quad u=U_e(x,t) \quad \text{and} \quad T\to T_\infty \qquad (15)$$

To complete the formulation of the problem, initial conditions must be specified in the (t, y) plane for $x=0$ and in the (x, y) plane for $t=0$. Consideration will be given here to a flow in which at time $t=0$, the flow field is given by steady-state conditions. At time $t>0$, the external velocity $U_e(x, t)$ begins to deviate from the steady-state velocity $U_0(x)$:

$$U_e(x,t)=U_0(x)G(t) \qquad (16)$$

where $G(t)$ represents unsteadiness of the free stream. It is assumed that $G(t)=1$ for $t<0$.

In order to evaluate the transient friction factor and heat transfer rates, Eqs (12), (13) and (14) have to be solved numerically to satisfy the appropriate initial and boundary conditions discussed previously. Specific forms of the transient function $G(t)$ have to be input. Since no experimental data for the title problem are reported in the literature concerning the details of $G(t)$, the numerical solution of Eqs (12), (13) and (14) is not undertaken. A discussion of the correlation of some experimental data reported by Base *et al*[20] for unsteady heat transfer will be provided in this paper.

Base *et al* performed an experimental study dealing with the combined effects of oscillating free stream velocity and free stream turbulence on the convective heat transfer from the stagnation point region on a circular cylinder in crossflow. In the wind tunnel which they used, the primary air flow to the inducer unit was varied with time so that the main flow in the wind tunnel working section also varied in a similar manner. Their results are shown in Fig 1 in which the effects of the harmonic stream

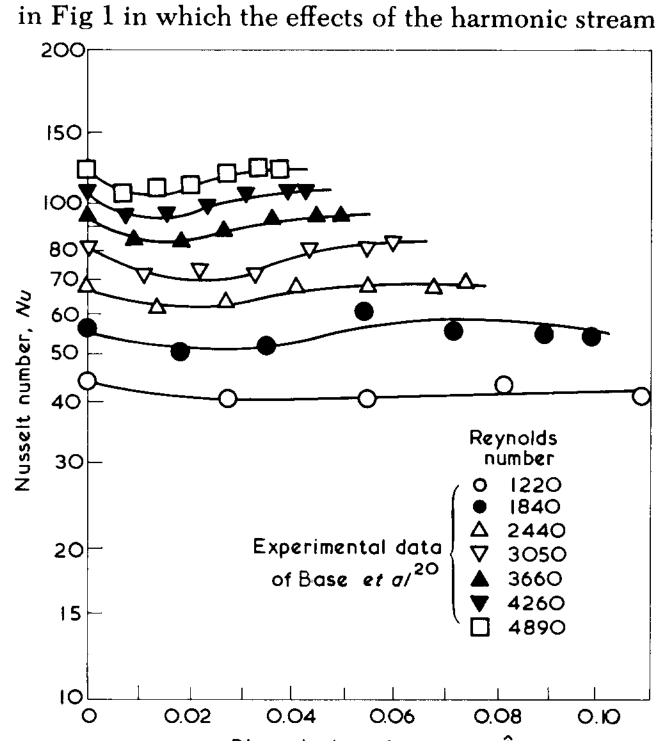

Fig 1 Nusselt number as a function of Strouhal number and Reynolds number

are plotted as the variation of Nusselt number with Reynolds number and reduced frequency. It may be seen that the Nusselt number increases with increasing values of Reynolds number at any given frequency of oscillation of the free stream.

It must be noted that Base *et al* could not obtain a suitable correlation for their results. An attempt will be made in this paper to do so. A dimensional analysis of the problem indicates that the Nusselt number is a function of three parameters, namely, Strouhal number ($\hat{f}$), turbulence intensity ($Tu = u'/U_\infty$) and the Reynolds number (Re). A unique grouping of these parameters has been found to correlate successfully with the Nusselt number. This will be discussed in the next section of the paper.

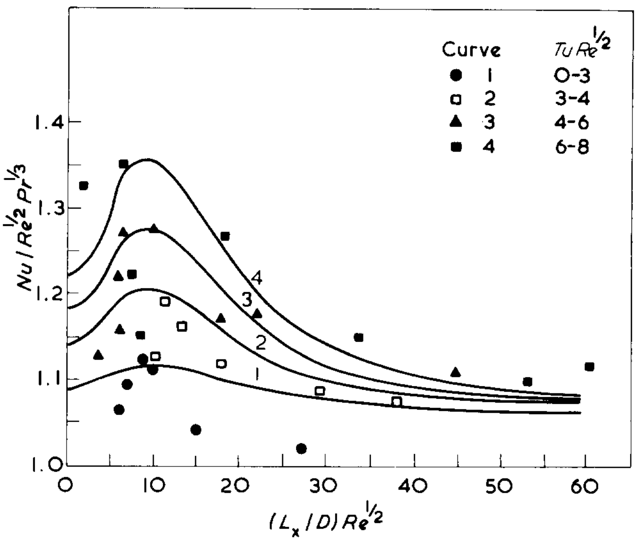

Fig 4 Effect of integral length scale on heat transfer

Results and discussion

Steady-state problem

A review of the literature suggests that there are several uncertainties in the published experimental data including tunnel blockage, variation of physical properties, effect of turbulence scale and inaccuracies in the measurement of turbulent intensity. Restricting the discussion to the stagnation point, it has been found convenient to use the single correlation parameter $TuRe^{1/2}$. This parameter has also been suggested by Smith and Kuethe[6] on the basis of a semi-empirical theory.

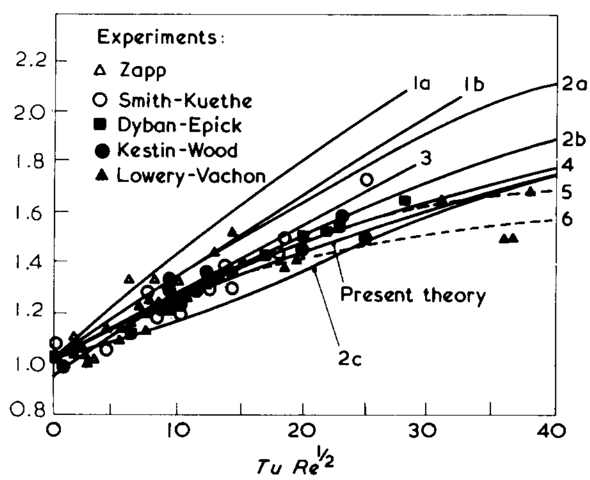

Fig 2 Nusselt number as a function of Tu $Re^{1/2}$

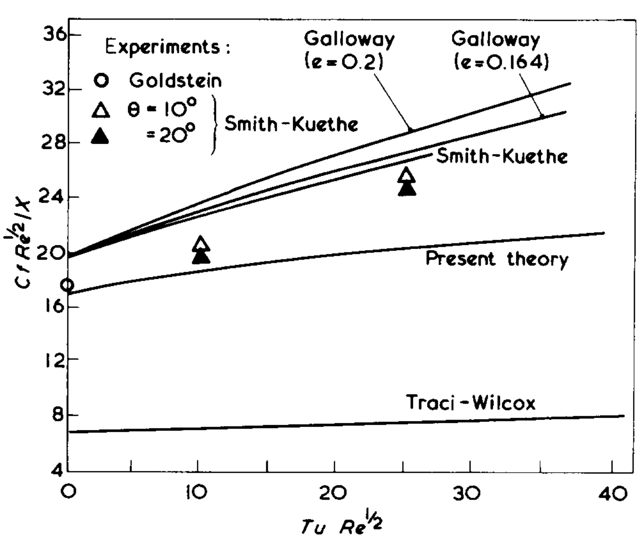

Fig 3 Skin friction coefficient as a function of Tu $Re^{1/2}$

Fig 2 shows the data from several sources in the published literature illustrating the variation of Nusselt number as a function of $TuRe^{1/2}$. The experimental data of Zapp[3], Smith and Kuethe[6], Dyban and Epick[8], Kestin and Wood[9] and Lowery and Vachone[10] indicate that $Nu/Re^{1/2}$ increases with $Tu\,Re^{1/2}$. In addition to the data from these experiments, curves depicting the predictions of Galloway (curves 1a, 1b), Traci and Wilcox (curves 2a, 2b and 2c), Smith and Kuethe (curve 3), Miyazaki and Sparrow (curve 4), Kestin and Wood (curve 5) and Lowery and Vachon (curve 6) are presented as continuous solid or dotted lines. In the present analysis, the constant s was adjusted so as to achieve the best fit of the present numerical predictions with experimental data available in the literature. It has been found that the following expression for s in Eq (6) shows the predictions of the present analysis to be in good agreement with the published data over the entire range of $Tu\,Re^{1/2}$:

$$s = 0.018 Tu\,Re^{1/2} \quad (17)$$

The experimental data indicate that $Nu/Re^{1/2}$ levels off at large values of $Tu\,Re^{1/2}$. When the free stream turbulence is increased, the boundary layer thickness also increases. The latter effect tends to neutralize the augmentation of the eddy diffusivity by the free stream turbulence thereby giving rise to the levelling-off of $Nu/Re^{1/2}$. The present analysis is seen to yield this behaviour quite satisfactorily.

Fig 3 shows the results for the skin friction coefficient as a function of the correlation parameter $Tu\,Re^{1/2}$. The results from the present analysis are seen to compare favourably with the experimental data of Smith and Kuethe[6]. In addition to the data of Smith and Kuethe, the predictive curves of Smith and Kuethe[6], Galloway[8] and Traci and Wilcox[17] are also shown in Fig 3.

Fig 4 illustrates the variation of the Nusselt number as a function of the dimensionless value of the integral length scale $(L_x/D)Re^{1/2}$ with the turbulence intensity parameter $Tu\,Re^{1/2}$ as a variable. The solid lines represent the predictions from the present analysis while the data points represent the experimental findings of Yardi and Sukhatme[21]. It has been found that the following expression for $\mathscr{F}(\bar{L})$ in Eq (6) makes the predictions of the present analysis to be in satisfactory agreement with the experimental results reported by Yardi and Sukhatme[21]:

$$\mathscr{F}(\bar{L}) = 1 + \frac{1.0593\bar{L}^{0.6042}}{\exp(0.642\bar{L})} \tag{18}$$

where

$$\bar{L} = (L_x/D)Re^{1/2}$$

The boundary layer thickness in the vicinity of the front stagnation point is proportional to $Re^{-1/2}$. Thus, $\bar{L}$ is proportional to the ratio of the integral length scale to the boundary-layer thickness and may be considered as a measure of the ability of the free stream turbulence to penetrate the boundary layer.

Fig 5 shows the variation of the skin friction coefficient as a function of the integral length scale $\bar{L}$, while the turbulence intensity parameter $Tu\,Re^{1/2}$ was chosen as a variable. The solid lines represent predictions from the present analysis. No experimental data exist in the published literature to compare with the present analytical predictions. The friction coefficient starts to increase with increasing values of the integral length scale parameter, goes through a maximum in the vicinity of $\bar{L} = 10$ and then tends to decrease with any further increase in $\bar{L}$.

Unsteady state problem

Fig 6 shows the correlation for the unsteady stagnation point heat transfer under the combined influence of free stream velocity fluctuations and turbulence intensity. The results reported by Base

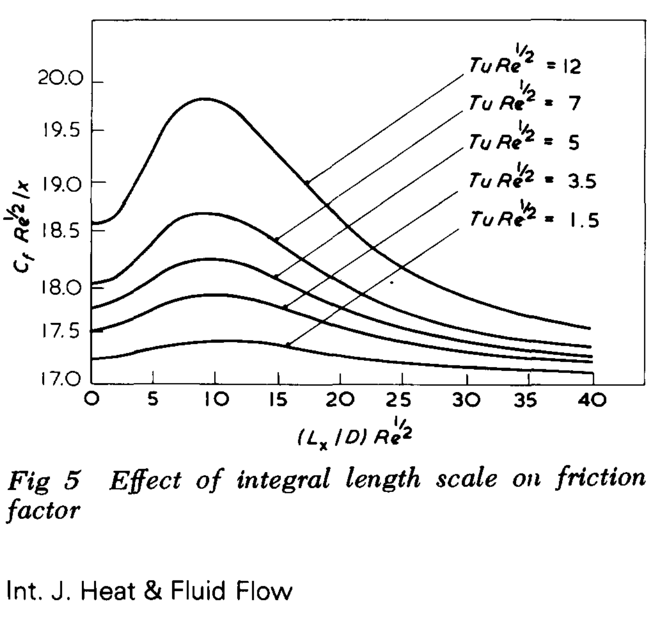

Fig 5 Effect of integral length scale on friction factor

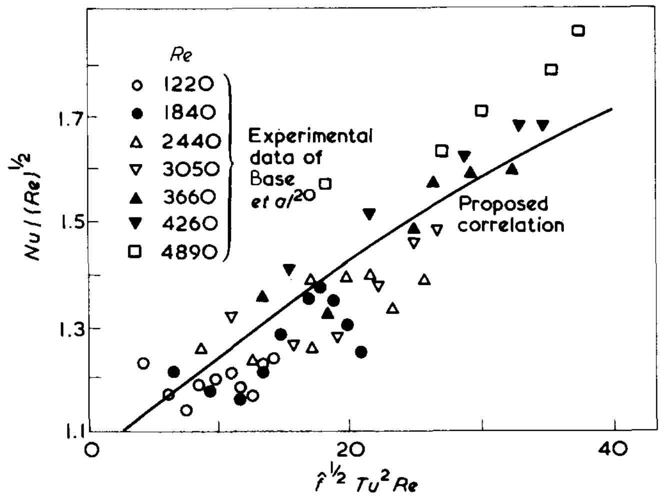

Fig 6 Correlation for heat transfer variation at the stagnation point region

et al[20] have been replotted in terms of $Nu/Re^{1/2}$ versus a new normalized turbulence parameter $\hat{f}^{1/2}\,Tu^2Re$. From the Figure it can be seen that the correlation is encouraging and suggests that there is a unifying trend. It has been found that the results for the normalized heat transfer with the new turbulence parameter introduced may be successfully correlated by the formula:

$$\frac{Nu}{Re^{1/2}} = 1.0521 + 0.0206[\hat{f}^{1/2}Tu^2Re] - (0.97\times10^{-4})[\hat{f}^{1/2}Tu^2Re]^2 \tag{19}$$

The above formula has been found to yield the best fit of the proposed curve to the experimental data reported in the literature by Base *et al*[20]. It should be noted that the Strouhal number $\hat{f}$ data were available only in Ref 20.

Conclusion

In this paper, an expression has been successfully formulated for the eddy diffusivity induced by the free stream turbulence intensity and integral length scale. The eddy diffusivity model was applied to the stagnation point of a cylinder situated in a uniform crossflow with free stream turbulence. The eddy diffusivity model included two prescribable parameters, namely, the turbulence intensity parameter $Tu\,Re^{1/2}$ and integral length scale parameter $(L_x/D)\,Re^{1/2}$. The solution of the steady-state momentum and energy equations with the proposed eddy diffusivity model yielded satisfactory predictions for the Nusselt number and skin friction coefficient in the presence of free stream turbulence intensity and integral length scale. These predictions have been compared with the published steady-state experimental and empirical data and agreement between the present analytical predictions and published data is seen to be very good considering the scatter in the experimental data points.

The experimental data reported in the literature concerning the unsteady stagnation point heat transfer under the combined influence of free stream velocity fluctuations and turbulence intensity have been analysed by dimensional analysis. A new turbulence parameter has been found and the unsteady heat transfer results have been successfully correlated by means of a second-order polynomial in terms of this new turbulence parameter.

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